

EXPERIMENT 2: FREE FALL ANALYSIS

Objective:

To determine gravitational acceleration by studying the velocity of a falling object as a function of time. A secondary objective is to evaluate the precision of your ruler-fit function, and to compare it to the “best-fit” function as determined by the Graphical Analysis program in the computer. By “precision” we mean the **root mean square** error denoted σ_{rms} .

Procedure:

Your lab instructor will demonstrate how to record the motion of a falling object using the tape timer. Work in groups of two, each student taking individual data with a different falling mass.

1. Cut a 1.5-meter length of recording tape. Make a small loop in one end and seal it with a piece of masking tape. Hang a 200-gram load from the loop. Trim the tape so that the free end will pass through the timer before the falling mass hits the floor.
2. Feed the other end of the tape up through the guides in the timer. Make sure the tape runs between the carbon paper and the strike plate. Pull up the free end of the tape until the falling mass is suspended just below the timer. The tape should be vertical and aligned with the guide slots to minimize drag as it is pulled through the timer.
3. With the tape vertical and with the weight stationary, switch the timer on at its 40-Hz pulse rate and release the tape. The timer will stamp spots on the tape giving the position of the falling mass at $1/40$ (0.025) second intervals.
4. With an unused tape and a different value for the falling mass (between 200 and 500 grams), record data for the other student in the group.

Analysis:

Look at your raw data; you will observe a track of spots on the paper tape. Since the separation between the points gets larger, you can see that the speed of the falling mass is increasing. To look for a more quantitative time-velocity relationship, you need to determine the speed of the falling body and plot a graph of speed vs time.

1. Pick a spot near the start of your data track and label it “ $t = 0$ ”. Start where the separation between spots is about 1cm; if less, the distance increments cannot be measured accurately. Label the spots consecutively with their times: $1/40$ s, $2/40$ s..., $13/40$ s. Measure and list in your data table the displacements from $t = 0$ to each later spot. Specify units and uncertainty. Also record the value of the falling mass. (you may use the table at the end of this section)

2. Instantaneous velocity. The instantaneous velocity, dx/dt , is equal to the average velocity $\Delta x/\Delta t$ over a 2/40 s interval **centered** at the time of interest. For example:

$$v_8 = \frac{x_9 - x_7}{t_9 - t_7}$$

Evaluate and add to your data table the instantaneous velocity (Be sure to use the distance between the *preceding* data point and the *following* data point.) Plot V(t) vs. time by hand. This is a full page graph with units and linear scales on the axes and bold data points. Don't forget to give the plot a title and make sure you label the axes (including units).

3. Acceleration. Use a ruler to fit a straight line through your time-velocity data Note: Your line will not pass through the origin . Evaluate the acceleration due to gravity by finding the slope of the line. What is the physical meaning of the y-intercept of this line? What is the percent difference between your result and the accepted value (9.81 m/s²)?
4. Computer fit. Now that you have drawn your best ruler-fit line through the data points, you can determine how precisely your line actually fits the data. Mathematically, the precision is described by giving the value of the root-mean-square standard deviation (σ_{rms}). This quantity is given by

$$\sigma_{rms} = \sqrt{\frac{\sum \delta_i^2}{N}} \quad \text{where } \delta_i = (V_i - (V_0 + at_i))$$

$i = 1, 2, \dots, 6.$

In our case, the relation between velocity and time is a straight line, as you can appreciate from the equation $V(t) = V_0 + at$. The deviation δ_i is the difference between the measured value V_i and the calculated value $V_0 + at_i$. From the equation above, you see that σ_{rms} is a measure of how well the straight line fits the group of data points. We will allow the computer to calculate σ_{rms} for us as it can easily find the best value for $V(t) = V_0 + at$. To do so, carry out the following steps:

Load the Windows program *Graphical Analysis* and enter your time and velocity data in columns. Switch off the "Connecting Lines" option in the Graph menu.

Select "Manual Curve Fit" from the analyze menu. Choose the linear function and enter your ruler-fit values in the slope and intercept boxes. The computer will draw the corresponding line and confirm your calculation of the mean squared deviation σ^2 . (This program reports σ^2 ; take the square root to find σ_{rms} .) Click the "OK – Keep Fit" button to add this line to your graph.

To have the computer find the best-fit to your data, select "automatic curve fit" from the Analyze menu; choose the linear function, and add this line to your graph with the "OK-Keep Fit" button. Print this graph, showing your velocity data with the ruler-fit and best-fit lines.

Uncertainty

The precision of the velocity measurement is given by σ_{rms} , which has dimension of velocity, and tells us the uncertainty for the velocity of at a typical data point. If we divide this by the

time interval over which our measurements were taken we have an estimate of the uncertainty in the value for g .

Report:

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

- 1) An appropriate title.
- 2) The Objective (One or two sentences).
- 3) Data table of time, distance and velocity; include data tape.
- 4) Hand plotted graph of velocity as a function of time.
- 5) Values for the acceleration and initial velocity as obtained from your graph.
- 6) A computer generated plot of your time-velocity data with the ruler fit and the computer generated fit. (Make sure the computer generated results are reasonable and have the right number of significant figures.)
- 7) A conclusion that compares the precision (σ_{rms}) and the accuracy (percent difference) from the accepted value for g for the two fits. Remember that precision indicates how well the best-fit line matches your data, while accuracy indicates how well the best fit line matches the accepted value (i.e. the textbook value) of g . Comment on why the computer fit has better precision than the hand fit. Do your measured values match the accepted value for g within the experimental uncertainty?

Data Table:

Data Point n	Time t_n	Distance x_n	$x_{n+1} - x_{n-1}$ Δx	Velocity $\Delta x / \Delta t$
	Units:	Units:	Units:	Units:
		$\pm$	$\pm$	
0	0	0		
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				

FALLING MASS: _____