Chapter 5 Jacobians: Velocities and Static Forces

Angular velocity of a rotating frame {B} in terms of {A}

 (*to denote the magnitude o*f ) (5.6)

Linear velocity of Q in frame {B} which is moving relative to {A}

 (5.7)

VQ

*K*



ΩB

Q

{B}

{A}

Rotational velocity of frame {B} with respect to {A}, AΩB, applied to AQB,

 , a vector cross product (5.10)

ΔQ = Qt sinθ ωΔt

AΩB

AVQ = AQB sinθ AΩB

Qt sinθ

Qt’

Qt

θ

With QB moving at velocity BVQ in {B} and frame {B} moving at AVBOrg with respect to {A}, the linear and rotational velocity of Q in moving and turning frame {B} with respect to {A},

 (5.13)

*Property of orthonormal rotation matrix for velocity analysis*

Taking a derivative of 

 (5.16)

So,  is a skew-symmetric matrix (in the form of *S + S-1 = 0*).

*Velocity of vector P due to rotating frame*

 (5.22)



, (5.24)

S is a skew-symmetric matrix (*S+S-1=0*)

*Skew-symmetric matrices and vector cross product*

If,=, and P=and then

= (5.27)

Then, from (5.24)

 (5.28)

From (2.80) and  for a small value of x according to the Taylor expansion,

 (5.33)

Taking the derivative w.r.t. θ

 (5.35)’

Transposing *R(t)* to the LHS, and recognizing the RHS as a skew symmetric matrix,

= (5.36)

where  (5.37)

Ω= angular velocity vector,

*K* = instantaneous axis of rotation

*Velocity Propagation for revolute joints – More equations!*

The angular velocity of link i+1 rotating about its Z axis, projected onto link i which also rotates,

 (5.44)

Multiplying both sides by 

 (5.45)

Linear velocity of frame {i+1}, dropping the last term as the frame origin is constant,

 (5.46)

Multiplying both sides by 

 (5.47)

*For prismatic joint (no rotational component in the frame, but a translational one.)*



 (5.48)

Jacobians

*Given a system of six non-linear equations for six X’s and six Y’s.*

*Y = F(X)*



 

A Jacobian in robotics is a matrix of partial derivatives that maps a joint velocity vector 

into a velocity vector of the end-effector:







 for six axis robots

*Changing a Jacobian reference frame in {B} to frame {A}:*

 (5.70)

Thus, (5.71)

Find and from (5.55) and (5.57)

*Singularities*

 (5.72)

If a matrix is singular, its determinant is zero, and so its inverse cannot be found. As such, under a singular condition, the end effector velocity cannot be translated into the joint angular velocity.

Types of singularities:

1. Work space boundary singularity – Occurs when the robot arm is fully stretched out with the end effector reaching the outer boundary.
2. Work space interior singularity – Occurs when two links line up to fold with the end effector inside the work space boundary.

*Static Forces in Manipulator*

****------ a cross vector product

ini = torque exerted on link i by link i-1, express in {i}

ifi = force exerted on link i by link i-1, express in {i}

*θ* i = angle between ifi and iηi

iPi+1 = displacement of link i+1, viewed in {i}

{i+1}

*iPi-1*

n**i+1**

{i}

f**i+1**

n**i**

f**i+1**

Equilibrium (counter balancing) of Force & Moment at a single link - Propagation Equations:

 (5.80)

 (5.81)

Joint torque at equilibrium –revolute joints = (Moment vector) (Joint axis vector)

 (5.82)

Joint torque at equilibrium –prismatic joints = (Force vector) (Joint axis vector)

 (5.83)

*The partial derivatives of (5.82) constitute a Jacobian. See Ex. 5.7.*

*Development of Jacobian for converting Force into Torque*

Work done in Cartesian space = Work done in Joint space

 (*6 x 1 vectors*) (5.91)

Rewriting in notation for matrix multiplication

 (5.94)

Since by definition,

 (5.95)

Since,



Transposing the two sides, 

 (5.96)

*A Jacobian transpose maps the gripper force into equivalent joint torques.*

*Force and Velocity Transformation in the tool frame*

{A}=Revolute, {B}=Fixed, per Fig. 5.13

(6x1) velocity vector: , and (6x1) force/moment vector: 

For the Velocity Transformation, starting from (5.45) and (5.47)

 (5.45)

 (5.47)

Setting…….(why?) and

 (5.101)



Note that

1. 
2. 



 (5.102)

The Force-Moment transformation is derived from (5.80) and (5.81):

 (5.80)

 (5.81)

 (5.105)

The relationship between Velocity transformation and Force-Moment transformation:

 (5.107)

Exercise 5.3

θ3

θ2

L1

θ1

L2

*Jacobian derived from the velocity propagation from Base to Tip*

Example 5.3: - RRR robot with frozen joint 3.

 are 3x1 vectors. The vector cross products in (5.47)  are shown below.

 

 (5.55)

 (5.56)

 (5.57)

Note that the components of may also be determined geometrically.

*Jacobian derived from Static Force propagation from Tip to Base*

*Jacobian derived from direct Differentiation of the kinematic equations*

By observation of the geometric link-frame diagram, the kinematic equations are:



Taking partial derivatives to arrive at a Jacobian,



Onceis found,can be found from:

,

whereis readily calculated from the rotational matrixes.