OPTIMIZATION OF FORMATION FLIGHT

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ABSTRACT

OPTIMIZATION OF FORMATION FLIGHT

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This report includes a literature search, an economic analysis, a design of the optimum formation flight and a study of the reduction of induced drag for different wing geometry. The project scope discusses the objectives of this report, and the approaches that will be taken to achieve the project's objectives. The literature survey compares the different kinds of numerical analyses and experiments. The economic analysis predicts the reduction of total jet fuel based on its price. Finally, the design stages are divided into the different wing shapes. All the simulations of the formation flight will be done using Matlab from Mathwork inc.

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Symbols

L lift
D drag
D _i induced drag
D ₀ profile drag
R range
W weight
q dynamic pressure
c_t thrust specific fuel consumption
Γcirculation
w down-wash
s span
S wing surface
ρ density

Chapter 1

Prologue

1.1 Motivation

There are two different types of jet aircraft fuel. The first one is JET A-1, which is the most common fuel. The other one is JET B, which is used in cold regions since the fuel has a lower flash point and freezing point. The price of jet fuel has increased 239% over in the last four years, and it is up to \$2.03 per gallon. The price is lower compared to car gas, but airplanes spend much more fuel than cars. An airline ticket from San Francisco to New York is around \$120. If the airline uses a 200-seat Boeing 757-200, the company will have to buy \$12,594 of jet fuel, and it will take 46% of the ticket sale. In these tough market conditions, recently two American major airlines, Delta and Northwest filed for bankruptcy. There may be several ways to reduce fuel consumption, such as cramping the passenger seats, buying brand new aircrafts or discovering new flight paths (?). There is another way to reduce fuel consumption without additional investment. That is formation flight. Jet fuel is burned to overcome the drag of aircrafts during flight. According to current research (Vachon, M. J. et al, 2003), formation flight can reduce the induced drag by 45% and the total drag by 16.5%. It means the airline company can save a maximum of \$2,078 from San Francisco to New York one way. Formation flight can also reduce the hazardous exhaust gas from the jet engines.

Even though there is possibility of 16.5% fuel savings, these benefits have not been gained. One reason is the lack of a sophisticated controller to keep appropriate distance between airplanes in the possible wake region. The other is the lack of sufficient research. Study has been defined a small number of airplane formation and a few wing geometries.

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There has not been in-depth research considering safety or optimal separation between aircrafts.

1.2 Objectives

This project has two objectives:

- (a) The first one is To find the optimal separation (lateral and longitudinal) for the most fuel efficient formation flight of jet transports.
- (b) The second one is To study the reduction of induced drag for different wing geometries.

Chapter 2

Introduction

Normally airfoils have more curvature on the upper surface than the lower surface. This shape tilts the wind direction, and the wind is called a *"relative wind,"* different from the free stream. Lift is expressed perpendicular to the wind direction, and drag is parallel to the wind direction.



Figure 2.1. Relative Wind over the airfoils.

Actual lift becomes perpendicular to the relative wind, so it tilts back compared to the original lift, which is perpendicular to the free stream direction. This tilting force is called *"induced drag"* and measured in the same direction as the free stream direction. The velocity vector of subtraction from the relative wind is called *"downwash."*



Figure 2.2 Downwash and upwash due to the leading airplane.

This vector is the multiplex of the induced angle of attack and free stream velocity in the small angle of attack. If the downwash is reduced, induced drag will be diminished. Lift is the pressure difference between the upper and lower surface of the wing. At the wing tip, air flows over the upper surface from the lower surface due to diffusion of air. This flow forms the vortex. This vortex contributes two different wind regions behind the airplane. The inside of the wing span has downwash, and the outside of the wing becomes the upwash region.



Figure 2.3 Down-wash and up-wash due to the leading airplane at one wing span behind



Figure 2.4 Induced drag, down-wash and up-wash.

A higher aspect ratio is favorable to produces less induced drag, but there is a structural limit and maneuvering restriction in that wing shape. For these reasons, the same effect is achieved by grouping the lower aspect ratio wings. In this solution the aircrafts are not located in the same longitudinal position since the group is not one rigid

body, even though it is one body aerodynamically. Practically, each airplane is set in different longitudinal locations and gets the benefit from the leading aircraft. If the aircraft fly in a group, the down-wash or up-wash will affect the trailing airplanes. When the following airplanes are located in the up-wash region of the leading aircrafts, their own down-wash can be diminished, and induced drag can be decreased at the same time.

There are two different formations, arrow and V-shaped. Figure 2.4. shows the relative position coordination in formation.



Figure 2.5 Relative coordination in formation What kind of formation is this? (Vachon, M. J. et al. 2003).

2.1 Background

Airplane drag plays an important role in performance, and performance is directly related to fuel consumption. Formation flight reduces the induced drag of trailing airplanes due to the up-wash from the leading aircrafts. With the current high oil price and increasing demand for air transportation, formation flight will save a large amount of fuel consumption. Despite the benefits of formation flight, currently there are only few optimization studies within a narrowly defined distance range using a limited number of airplanes and wing geometries. Optimization of the formation flight in a large number of airplanes and wing shapes will give a practical solution. If this solution includes considering the aircraft's rolling and yawing space, wing shape and chamber, it will answer the safety issue.

2.2 Current Experiments

There are several studies on formation flight and control. One of them is the F/A-18 Performance report (Vachon, M. J. et al. 2003). It focuses on the optimum distance for maximum induced drag. There were two airplanes in various vertical and lateral positions from half to seven times the wing span distance from the leading airplane. The flight condition was Mach 0.8 at 36,000 ft altitude. The F/A-18 has a smaller aspect ratio of wing compared to commercial aircraft and a lower aspect wing in front. It is equipped with two tail wings, but it might not affect the up-wash flow in the experiment. This report shows the maximum induced drag at three to four times the wing span separation in a longitudinal direction and one tenth the wing tip overlap position in a lateral direction. The reduction of induced drag at this position is 48 percent, and total drag reduction is 20 percent.

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2.3 Current Analyses

Although there are few experiments, many theoretical papers exist. Two papers: one is written by Fiefel (Feifel, W. M., 1976) and the other by Maskew (Maskew, B., 1977). The both authors were NASA employees and the results were published in mid-70s. They applied the vortex lattice method. Fiefel used five airplanes in a V-shaped setting and showed the induced drag reduction in one case. Maskew used three aircrafts in an arrowshaped formation. He expressed the reduction of induced drag in the function of three dimensional distances. At the end of the 90s, Gingras (Gingras, D. R., 1999) experimented the formation in the wind tunnel, and UCLA students studied radiocontrolled airplanes in formation.

As mentioned in the paper (Chichka, F. D and Speyer, L. J., 1999), there is no study which considers the wing thickness and different wing geometries. There is neither the optimization in V-shaped formation nor study on a large number of fleets.

2.4 Current Computer Codes

NASA developed a FORTRAN code for VLM (Margason & Lamar, 1971). There is also the latest Matlab program, Tornado, written by Tomas Melin at the Royal Institute of Technology in 2001. This program uses VLM and calculates the lift, moment and down-wash behind the wing. Although it can handle the reflected flap, the wing is modeled as a flat panel. Both programs handle only one airplane.

Chapter 3

The Model of Air Flow

3.1 How Vortex Lattice Method Works

According to the Kutta-Joukowski theorem, circulation produces is the main force of lift. (Circulation is not a force!) On the other hand, the velocity difference between the upper surface and the lower surface of the wing generates lift. A vortex can mathematically show the jump in velocity when this occurs. Due to this characteristic, a group of vortices is used to show circulation around the wing and to estimate the lift by the wing.

$$L = \rho_{\infty} V_{\infty} \Gamma \tag{3.1}$$



Figure 3.1. The Arrangement of Vortices over the Airfoils.

Two boundary conditions should be satisfied to determine the strength of each vortex. The first condition is tangency condition at each control point, and the second is the Kutta condition at the trailing edge.

The vortex lattice method is simply covering the wing with many vortex lattices. Each vortex lattice is overlapped chord wise but not span wise. Normally the leading edge vortex is set at the quarter of each chord. The strength of each vortex is calculated by the tangency condition. This boundary condition is applied in each control point which is three fourths of the distance from the leading edge on the wing.



Figure 3.2. The Arrangement of Vortices over the Wings in the Vortex Lattice Method



Figure 3.3. Biot-Savart Law.

$$d\vec{V} = \frac{\Gamma_n(d\vec{l} \times \vec{r})}{4\pi r^3}$$
(3.2)

This equation shows the velocity at a certain distance, r, which is induced by a vortex filament and a vortex string length. The total velocity due to one vortex lattice is the sum of velocity by three filaments. The magnitude of down-wash can be given by vector dot operation with a vertical direction unit vector.



Figure 3.4. Coordination of the Vortex Lattices.

The general down-wash expression in the space due to a vortex lattice is composed of these three equations.

$$\mathbf{V}_{AB} = \frac{\frac{\Gamma_n}{4\pi} \frac{(x - x_{1n})(y - y_{2n}) - (x - x_{2n})(y - y_{1n})}{[(x - x_{1n})(y - y_{2n}) - (x - x_{2n})(y - y_{1n})]^2 + [(x - x_{1n})(z - z_{2n}) - (x - x_{2n})(z - z_{1n})]^2 + [(z - z_{1n})(y - y_{2n}) - (z - z_{2n})(y - y_{1n})]^2}$$

$$\begin{bmatrix} (x_{2n} - x_{1n})(x - x_{1n}) + (y_{2n} - y_{1n})(y - y_{1n}) + (z_{2n} - z_{1n})(z - z_{1n}) \\ \hline \sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2} \\ (x_{2n} - x_{1n})(x - x_{2n}) + (y_{2n} - y_{2n})(y - y_{2n}) + (z_{2n} - z_{1n})(z - z_{2n}) \\ \hline \sqrt{(x - x_{2n})^2 + (y - y_{2n})^2 + (z - z_{2n})^2} \end{bmatrix} (3.1)$$

$$V_{A\infty} = \frac{\Gamma_n}{4\pi} \frac{(-y + y_{1n})}{(z - z_{1n})^2 + (-y + y_{1n})^2} \left[1 + \frac{(x - x_{1n})}{\sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}} \right] (3.2)$$

$$V_{B\infty} = -\frac{\Gamma_n}{4\pi} \frac{(-y+y_{2n})}{(z-z_{2n})^2 + (-y+y_{2n})^2} \left[1 + \frac{(x-x_{2n})}{\sqrt{(x-x_{2n})^2 + (y-y_{2n})^2 + (z-z_{2n})^2}} \right] (3.3)$$

The total down-wash at the point is the sum of these three velocity components. The tangency equation originally had down-wash and a free stream value. Trailing airplanes additionally have up-wash from the leading airplane. If up-wash is considered in the tangency condition equation of the trailing airplane, a new circulation value or induced drag for each control point can be calculated.

$$D_{i} = -\int_{-s}^{s} \rho_{\infty} \left(w_{TrailingAirplane} - w_{LeadingAirplane} \right) \Gamma dy$$
(3.7)

The new values are smaller, which means less induced drag.

3.2 Flight Range

Jet aircraft flight range is the function of thrust-specific fuel consumption, lift and drag coefficients.

$$R = 2\sqrt{\frac{2}{\rho_{\infty}S}} \frac{1}{c_t} \frac{\sqrt{C_L}}{C_D} \left(\sqrt{W_{initial}} - \sqrt{W_{final}} \right)$$
(3.4)

The drag coefficient is composed of profile drag and induced drag coefficients.

$$C_D = C_{D,0} + C_{D,i} \tag{3.5}$$

The Induced drag coefficient is simply dividing induced drag by dynamic pressure and a wing surface.

$$C_{D,i} = \frac{D_i}{q_{\infty}S}$$
(3.6)

The induced drag is the function of down-wash.

$$D_i = -\int_{-s}^{s} \rho_{\infty} w \Gamma dy \tag{3.7}$$

Chapter 4

The Vortex Lattice Method on a Flat Plate

4.1 Programming for Reduction of Induced Drag

Currently the program only calculates the reduction of induce drag of one aircraft in the formation flight. To find the total reduction, the program should be run repeatedly for each airplane.

4.1.a. Program Coding

The program calculates down-wash, induced drag and flight range. It has four subfunctions: drawing wing geometry, setting control points, calculating down-wash and figuring out the induced drag and flight range.





This version of the program uses a thin plate as an airfoil of the wing and small degree angle of attack approximation. The flow analysis is done in the region including the outside of the wing from the leading edge. The next version inputs will be airfoil type, the section of the airfoil and twist angle. For the formation setting, the coordination of trailing airplanes in three dimensions is the only input. Later, this program is going to be applied to the control points on the mean camber line to capture the cambered airfoil.

The camber thickness is included in the second version of the program. The wing geometry becomes 3D like with expanded panel airfoil sections. Making the second version will be easier than the current version because the leading edge positions for each airfoil are already set, so the different thicknesses can be just added. The geometry drawing is not hard, but two things are difficult to handle in the program. The first one is applying the Kutta condition at the trailing edge. The second one is considering different stagnation points at the near leading edge. Both challenges will come from the twist angle. After completing the three-dimensional wing shape, the simple addition of each down-wash at the control point will shows the reduction of induced drag.

4.1.b. Program Accuracy

Before applying the program to optimization of formation flight, the program performance is checked with published theoretical result (Bertin, 2002, p. 273) and experimental data (Vachon et al, 2003, p. 19).

8 Vortex Lattices	Wing Geometry			Circulat	ion Stren	gth	
(4 on symmetric left and right wing)				(Γ_n/bU_{∞})	α)		
	Sweep Angle (degree)	Taper ratio	Aspect ratio	Panel 1	2	3	4
Theory	45	1	5	0.0273	0.0287	0.0286	0.0250
Program				0.0273	0.0287	0.0286	0.0250

Table 4.1 Comparison between theoretical and program result.

The results are matched.

8 Vortex	Aircraft	Separation	Induced	Induced
Lattices	Туре	(in span)	Drag	Drag
Formation Flight of Two-		(in span)	(Newton)	Reduction (percent)

Airplane		Longitudinal	Lateral	Vertical		
Experiment	F/A-18	3.0	-0.05	-0.15	NA	45
Program		1.0	-0.05	0.00	307	46
Experiment		1.5	-0.21	0.00	NA	46
Program]					

Table 4.2. Comparison between Experimental and Program Result.

There is a difference between the results. I am reviewing the code, and the increase of the vortex lattice number in chord wise can reduce the difference.

Chapter 5

Implementation of a Vortex Lattice Method

5.1 Project Scope and Specification

This project optimizes the separation in formation analytically and numerically. An optimization approach is at most one foot behind a leading aircraft. Vortex Lattice Method is a lift and down-wash calculation tool. It is faster than Vortex Panel Method but can be less accurate. There are several versions. The goal of optimization formation flight is saving jet fuel. The functional requirement of the fuel consumption and flight range are operated in an environment with an average temperature at the altitude.

Other main specifications and parameters for the aircraft of formation flight are listed below.

	Boeing 747-400 (Boeing co., N.D.)	F/A-18 (Pike, 2005)
Cruise speed (Mach)	0.855 (567 mi/h 912 km/h)	0.8
Main wing span (ft)	211	37 (11.43 m)
Main wing surface (ft ²)	(524.9 m^2)	(37.16 m^2)
Length (ft)	229	56 (17.07 m)
Fuel tank capacitance (gal)	57,285 (216,840 L)	1,304 (4,936 L)
Gross weight (lb)	875,000 (396,900 kg)	36,708 (16,651 kg)
Range (nmi)	7,260 (13,450 km)	1,800 (3,333 km)
Fuel consumption (lb/nmi)	7.89 (16.12 L/km)	0.72 (1.48 L/km)

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5.2 Results

There are two different formations, arrow and V-shaped. Figure 2.4. shows the relative position coordination in formation.



Figure 5.1 Relative coordination in formation (Vachon, M. J. et al. 2003).

5.2.a. Two-airplane arrow-shaped formation

8 Vortex	Aircraft	Separation (in span)	Induced	Induced Drag	Lift
Lattices	Type		Drag	Coefficient	Coefficient
		(iii spaii)			

		longitu dinal	Lateral	Vertical	(Newton)		
Solo Flight	Boeing 747-400	NA			56,126	0.0059	0.1679
	F/A-18	NA			575	0.0052	0.1490
Formation Flight of Two-	Boeing 747-400	1	0.50	0	54,809	0.0057	0.1676
Airplane	F/A-18	1	-0.05	0	307	0.0028	0.1490

Table 5.2. Comparison of Aerodynamic Characteristics for Two degree Angle of Attack

Down-wash at each control point: 6.6874 m/s and F/A-18 2.9089 m/s

Up-wash due to the leading airplane at the trailing airplane control point in m/s:

1) Starboard

Boeing 747-400 at 1/1.5/0

Cp 1	Cp 2	Ср 3	Cp 4
0.1124	0.1415	0.1846	0.2493

Boeing 747-400 at 1/-0.05/0

Cp 1	Cp 2	Ср 3	Cp 4
0.2174	0.3530	0.7206	7.4646

F/A-18 at 1/-0.05/0

Cp 1	Cp 2	Ср 3	Cp 4
0.3177	0.5146	1.0654	11.5091

2) Port

Boeing 747-400 at 1/-0.05/0

Cp 1	Cp 2	Ср 3	Cp 4
0.1492	0.1099	0.0845	0.0671

F/A-18 at 1/-0.05/0

Cp 1	Cp 2	Cp 3	Cp 4
0.2177	0.1593	0.1217	0.0961

5.2.b Three-airplane V-shaped formation

This case study uses three airplanes, and achieves an average of 8.23% total induced drag and 6.74% total drag reduction. The range also extends by 6.74% (14,703 \rightarrow 15,694km). This result means fuel consumption can be decreased by 6.74% (1.08 liter saving for each kilo-meter and total 14,615 liter saving).

8 Vortex Lattices		Separation (in span)			Induced Drag (Newton) Change in Percent	Induced Drag Coefficient	Lift Coefficient
		Longitu dinal	Lateral	Vertical			
Solo Flight	Boeing 747- 400				56,126	0.0056	0.1679
Formatio n Flight of Five-	1 st Aircraft	0	0	0	54,969 -2.06 %	0.0055	0.1679
Airplane	2 nd	1	-1	0	49,772 -11.32 %	0.0050	0.1679

8 Vortex Lattices		Separation (in span)			Induced Drag (Newton) Change in Percent	Induced Drag Coefficient	Lift Coefficient
		Longitu dinal	Lateral	Vertical			
	3 rd	1	0	0	49,772 -11.32 %	0.0050	0.1679

Table 5.3 Comparison of aerodynamic characteristics for two degrees angle of attack

5.2.c. Five-airplane V-shaped formation

This case study uses five airplanes, and achieves an average of 10.40% total induced drag and 7.7% total drag reduction. The range also extends by 7.7% (14,703 \rightarrow 15,835 km). This result means fuel consumption can be decreased by 7.7% (1.24 liter saving for each kilo-meter and total 16,696 liter saving).



Figure 5.2 Five-airplane V-shaped formation.



Figure 5.3 Down-wash contributions at each control point of the 2nd airplane.

8 Vortex Lattices		Separatic (in span)	'n		Induced Drag Change in Percent	Induced Drag Coefficient	Lift Coefficient
		Longitudi nal	Lateral	Vertical			
Solo Flight	Boeing 747- 400				56,126 N	0.0059	0.1679
Formatio n Flight of Five- Airplane	1 st Aircraft	0	0	0	54,809 N -2.41 %	0.0058	0.1679
	2 nd	1	-1	0	-12.44 %	0.0052	0.1679
	3 rd	1	0	0	-12.44 %	0.0052	0.1679
	4 th	2	-2	0	-12.31 %	0.0052	0.1679
	5 th	2	1	0	-12.31 %	0.0052	0.1679

Table 5.4 Comparison of aerodynamic characteristics for two degrees angle of attack

There is a trial of three different cases to find the maximum reduction of induced drag of the fourth airplane. The positions of the other airplanes are set the same relative to the leading airplane. This variance of position and the reduction are shown in Table 5.3.

8 Vortex Lattices		Separation (in span)			Induced Drag Change in Percent	Induced Drag Coefficient	Lift Coefficient
		Longit udinal	Lateral	Vertical			
Solo Flight Formatio n Flight of Five- Airplane	Boeing 747-400	NA	•	•	NA	0.0056	0.1679
		2	-1.96	0	-21 %	0.0044	0.1679
		2	-1.95	0	-30 %	0.0039	0.1679
		2	-1.94	0	-112 %	-0.0007	0.1679

Table 5.5 Comparison of reduction of the induced drag of 4th airplane.

In the third case, the induced drag reduction is over 100%. It means that the up-wash due to the other airplanes is larger than its down-wash. It extends the total range of 4^{th} airplane by 9.3% (14,730 \rightarrow 16,103 km).

Around the leading airplanes' wing tips, the reduction of induced drag and flight range are larger and more abrupt than other position. The changes seem non linear variation.



Figure 5.4 Change of induced drag and flight range at different positions of 4th airplane in

the five-airplane formation.

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Appendix

A.1. Program Source Code