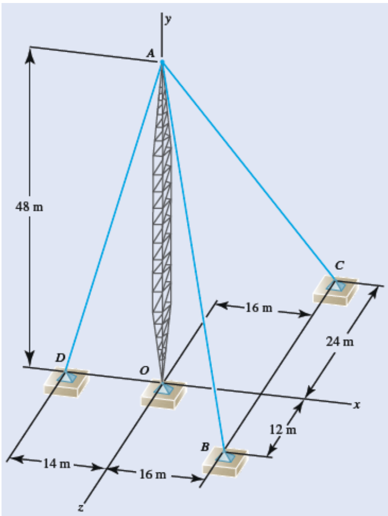
Solution 3.38



PROBLEM STATEMENT

Three cables are attached to the top of the tower at A. Determine the angle formed by cables AD and AB.

First note:

$$\overline{AB} = (16\text{ m})\mathbf{i} - (48\text{ m})\mathbf{j} + (12\text{ m})\mathbf{k}$$
$$\overline{AD} = -(14\text{ m})\mathbf{i} - (48\text{ m})\mathbf{j}$$

$$AB = \sqrt{(16\text{ m})^2 + (-48\text{ m})^2 + (12\text{ m})^2} = 52.0\text{ m}$$

$$AD = \sqrt{(-14\text{ m})^2 + (-48\text{ m})^2} = 50.0\text{ m}$$

By definition,

$$\overline{AB} \cdot \overline{AD} = (AB)(AD)\cos\sphericalangle BAD$$

$$\cos\sphericalangle BAD = \frac{\overline{AB} \cdot \overline{AD}}{(AB)(AD)}$$

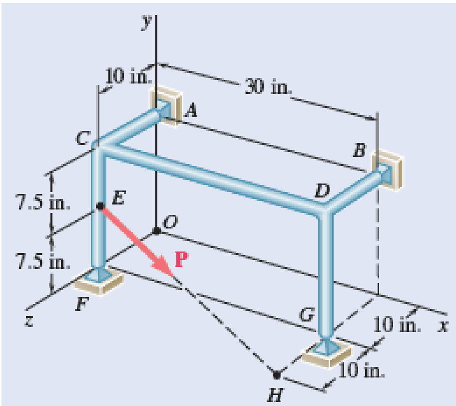
$$\cos\sphericalangle BAD = \frac{\left((16\text{ m})\mathbf{i} - (48\text{ m})\mathbf{j} + (12\text{ m})\mathbf{k}\right) \cdot \left((-14\text{ m})\mathbf{i} - (48\text{ m})\mathbf{j}\right)}{(52.0\text{ m})(50.0\text{ m})}$$

$$\cos\sphericalangle BAD = \frac{(16)(-14) + (48)^2}{(52.0)(50.0)}$$

$$\cos\sphericalangle BAD = 0.800$$

$$\theta = 36.9° \blacktriangleleft$$

Solution 3.55



PROBLEM STATEMENT

A force **P** of magnitude 520 lb acts on the frame shown at point *E*. Determine the moment of **P** about a line joining points *O* and *D*.

First develop expression for *P* in terms of its unit vector:

$$\overrightarrow{EH} = (30 \text{ in.})\mathbf{i} - (7.5 \text{ in.})\mathbf{j} + (10 \text{ in.})\mathbf{k}$$

$$EH = \sqrt{(30)^2 + (-7.5)^2 + (10)^2} = 32.50 \text{ in.}$$

$$\lambda_{EH} = \frac{\overrightarrow{EH}}{EH} = \frac{30\mathbf{i} - 7.5\mathbf{j} + 10\mathbf{k}}{32.50}$$

$$\lambda_{EH} = 0.92308\mathbf{i} - 0.23077\mathbf{j} + 0.30769\mathbf{k}$$

$$\mathbf{P} = P\lambda_{EH} = 520 \text{ lb}\left(0.92308\mathbf{i} - 0.23077\mathbf{j} + 0.30770\mathbf{k}\right)$$

$$\mathbf{P} = (480.0 \text{ lb})\mathbf{i} - (120.0 \text{ lb})\mathbf{j} + (160.0 \text{ lb})\mathbf{k}$$

The moment of **P** about axis *OD* is given by the mixed triple product Eq. (3.43):

$$\mathbf{M}_{OD} = \lambda_{OD} \cdot (\mathbf{r}_{E/D} \times P)$$

$$\mathbf{r}_{E/D} = -(30 \text{ in.})\mathbf{i} - (7.5 \text{ in.})\mathbf{j}$$

$$\lambda_{OD} = \frac{\overrightarrow{OD}}{OD} = \frac{30\mathbf{i} + 15\mathbf{j} + 10\mathbf{k}}{35.0}$$

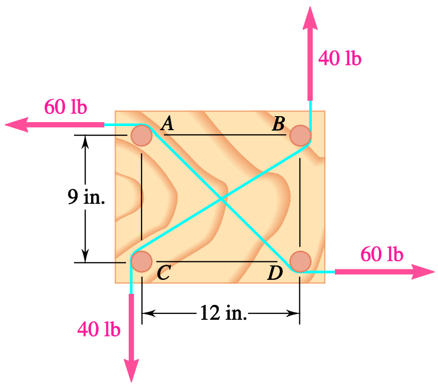
$$\lambda_{OD} = 0.85714\mathbf{i} + 0.42857\mathbf{j} + 0.28571\mathbf{k}$$

$$\mathbf{M}_{OD} = \lambda_{OD} \cdot (\mathbf{r}_{E/D} \times P) = \begin{vmatrix} 0.85714 & 0.42857 & 0.28571 \\ -30.0 \text{ in.} & -7.50 \text{ in.} & 0 \\ 480 \text{ lb} & -120 \text{ lb} & 160 \text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{OD} &= 0.85714[(-7.50 \text{ in.})(160 \text{ lb}) - 0] \\ &\quad + 0.42857[0 - (-30.0 \text{ in.})(160 \text{ lb})] \\ &\quad + 0.28571[(-30.0 \text{ in.})(-120 \text{ lb}) - (-7.50)(480)] \\ &= +3085.7 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$\mathbf{M}_{OD} = +3090 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

Solution 3.73



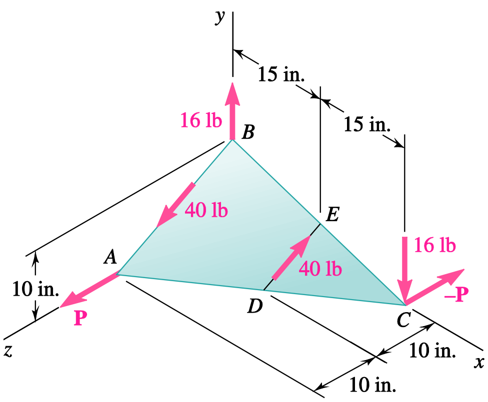
PROBLEM STATEMENT

Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 1132.5 lb·in. counterclockwise.

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$

$$1132.5 \text{ lb}\cdot\text{in.} = [(9+d)\text{ in.}](60\text{ lb}) + [(12+d)\text{ in.}](40\text{ lb}) \qquad d = 1.125\text{ in.} \blacktriangleleft$$

Solution 3.76



PROBLEM STATEMENT

If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2; \quad F_1 = 16 \text{ lb}, \quad F_2 = 40 \text{ lb}$$

$$\mathbf{M_1} = \mathbf{r}_C \times \mathbf{F}_1 = (30 \text{ in.})\mathbf{i} \times [-(16 \text{ lb})\mathbf{j}] = -(480 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\mathbf{M_2} = \mathbf{r}_{E/B} \times \mathbf{F}_2; \quad \mathbf{r}_{E/B} = (15 \text{ in.})\mathbf{i} - (5 \text{ in.})\mathbf{j}$$

$$d_{DE} = \sqrt{(0)^2 + (5)^2 + (10)^2} = 5\sqrt{5} \text{ in.}$$

$$F_2 = \frac{40 \text{ lb}}{5\sqrt{5}}(5\mathbf{j} - 10\mathbf{k})$$

$$= 8\sqrt{5}[(1 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}]$$

$$\mathbf{M}_2 = 8\sqrt{5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$\mathbf{M} = -(480 \text{ lb}\cdot\text{in.})\mathbf{k} + 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$= (178.885 \text{ lb}\cdot\text{in.})\mathbf{i} + (536.66 \text{ lb}\cdot\text{in.})\mathbf{j} - (211.67 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$$

$$= 603.99 \text{ lb}\cdot\text{in}$$

$M = 604 \text{ lb}\cdot\text{in.}$ ◀

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.29617\mathbf{i} + 0.88852\mathbf{j} - 0.35045\mathbf{k}$$

$$\cos\theta_x = 0.29617$$

$$\cos\theta_y = 0.88852$$

$$\cos\theta_z = -0.35045$$

$\theta_x = 72.8° \quad \theta_y = 27.3° \quad \theta_z = 110.5°$ ◀