Analysis of Statically Indeterminate Structures Using the Force Method

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Statically Indeterminate Structures

At the beginning of the course, we learned that a **stable structure** that contains **more unknowns than independent equations of equilibrium** is **Statically Indeterminate.**

Advantages

- Redundancy (several members must fail for the structure to become unstable);
- Often maximum stresses is certain members are reduced;
- Usually deflections are reduced.

Disadvantages

- Connections are often more expensive;
- Finding forces and deflections using hand analysis is much more complicated.

Steps in Solving an Indeterminate Structure using the Force Method

Determine degree of Indeterminacy
Let *n* =degree of indeterminacy
(i.e. the structure is indeterminate to the nth degree)

Chapter 3

Define Primary Structure and the *n* Redundants

Define the Primary Problem

Solve for the *n* Relevant Deflections in Primary Problem

Chapters 3,4,5 then 7 or 8

Define the *n* Redundant Problems

Solve for the *n* Relevant Deflections in each Redundant Problem

Chapters 3,4,5 then 7 or 8

Write the *n* Compatibility Equations at Relevant Points

Solve the *n* Compatibility Equations to find the *n* Redundants

Use the Equations of Equilibrium to solve for the remaining unknowns

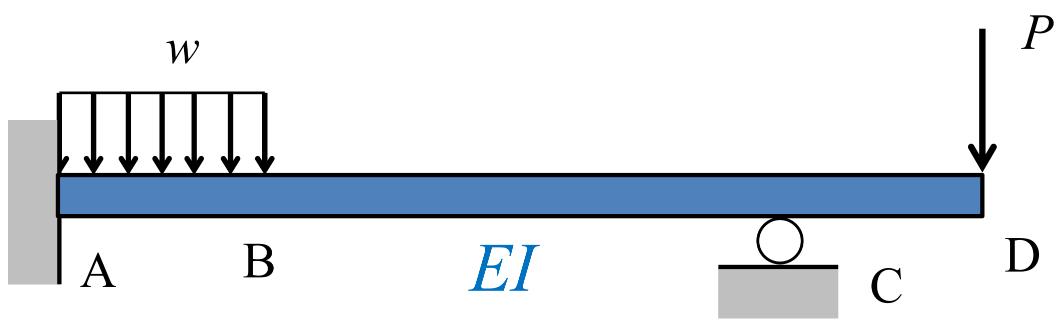
Chapter 3

Construct Internal Force Diagrams (if necessary)

Chapters 3,4,5

Force Method of Analysis

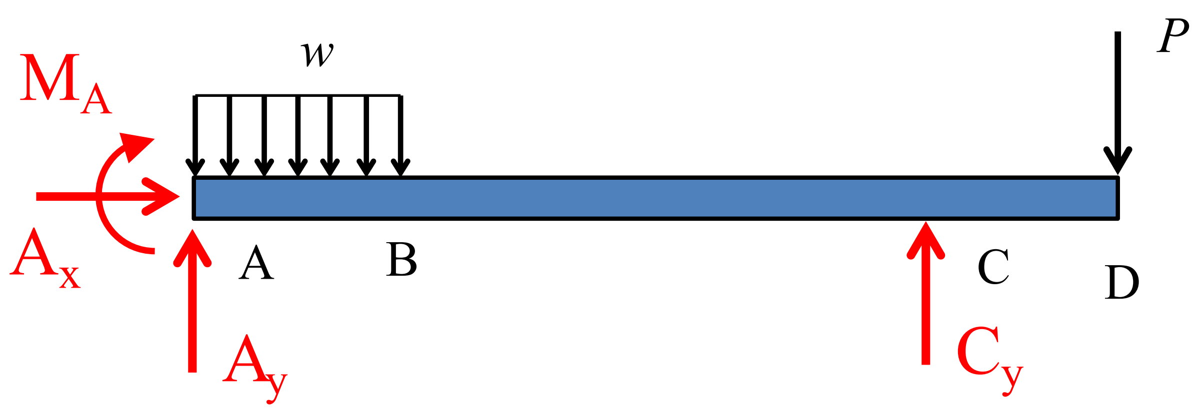
Consider the beam



Beam is stable

FBD

$X = 4$



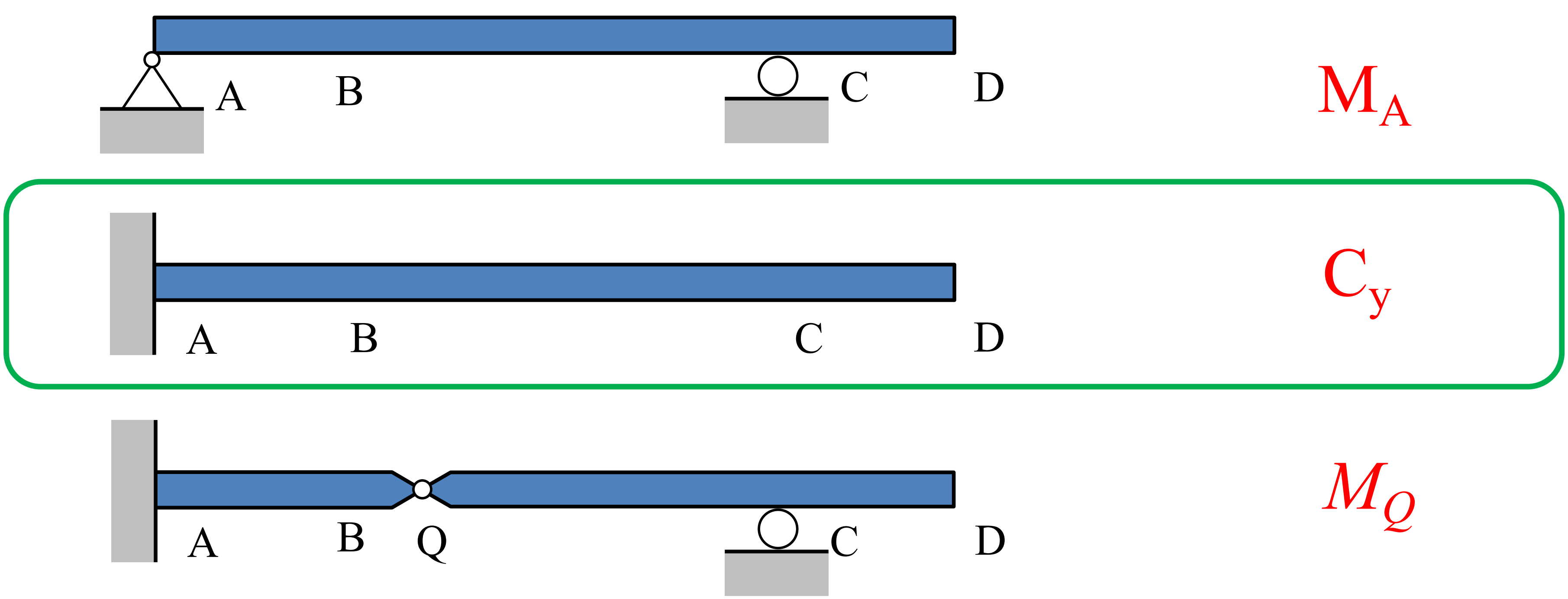
$3n = 3(1) = 3$

Statically Indeterminate to the 1st degree

Define Primary Structure and Redundants

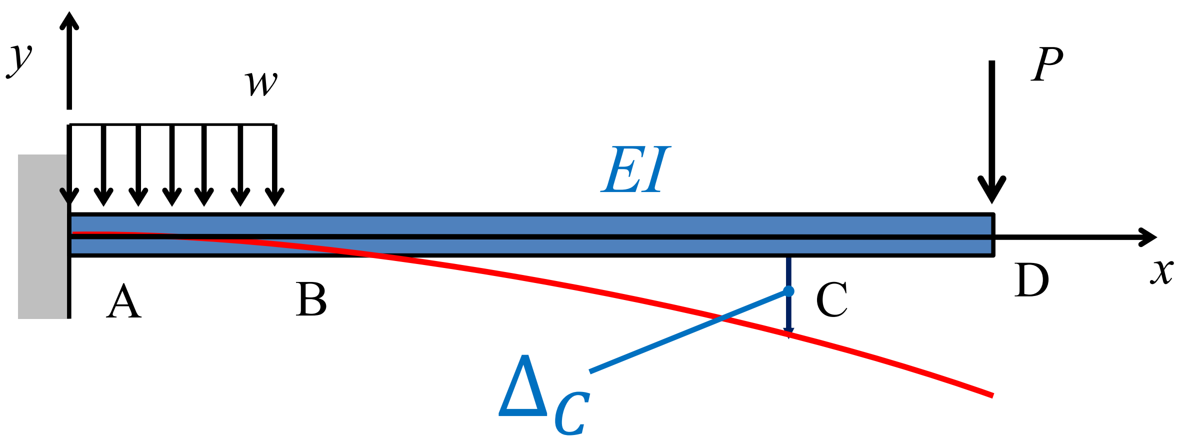
- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.

Primary Structure | **Redundant**



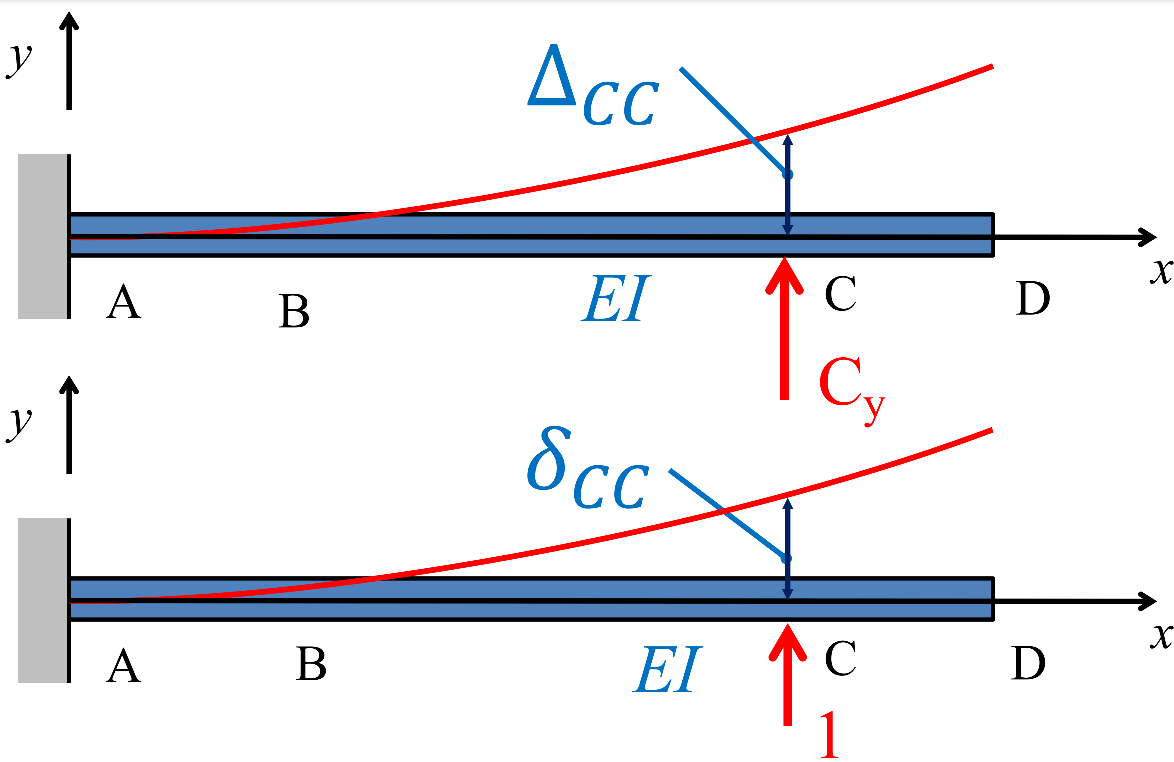
Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.



Define and Solve the Redundant Problem

- There are the same number of redundant problems as degrees of indeterminacy;
- Define a reference coordinate system;
- Apply only one redundant to the primary structure;
- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;



Redundant Problem

$$\Delta_{CC} = C_y \delta_{CC}$$

Compatibility Equation

Compatibility at Point C

$$\Delta_C + \Delta_{CC} = 0$$

$$\Delta_C + C_y \delta_{CC} = 0$$

Solve for C_y

The Force Method is Based on the Principle of Superposition

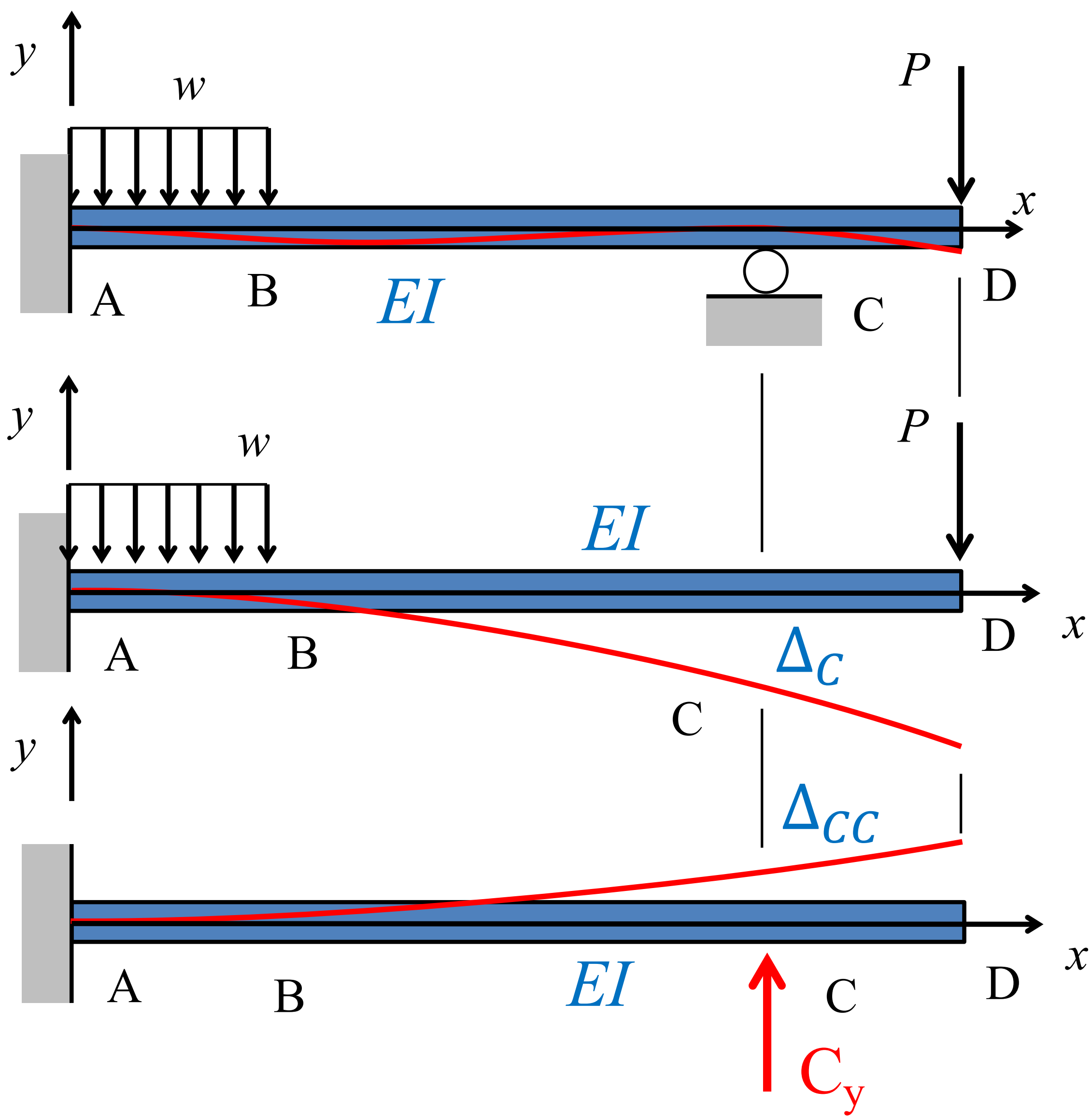
Indeterminate Problem

=

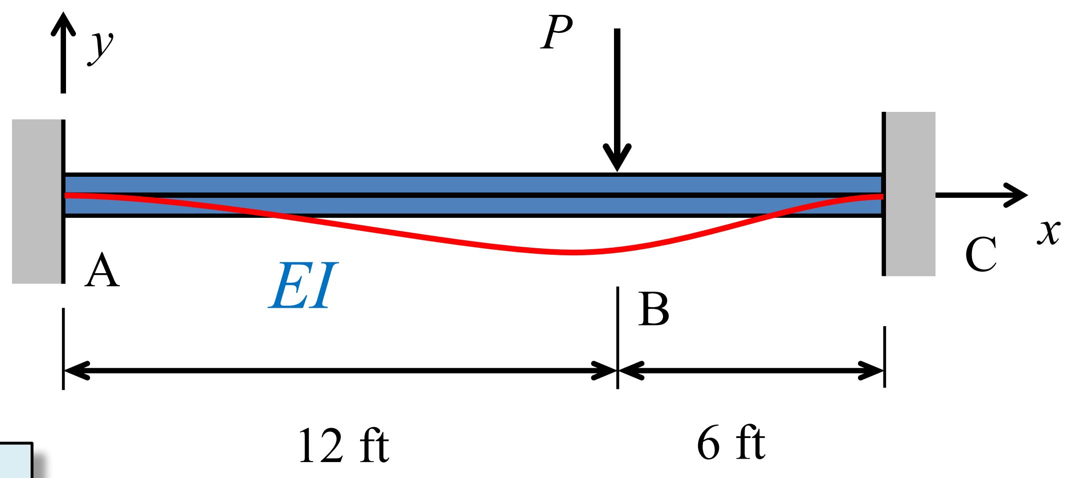
Primary Problem

+

Redundant Problem

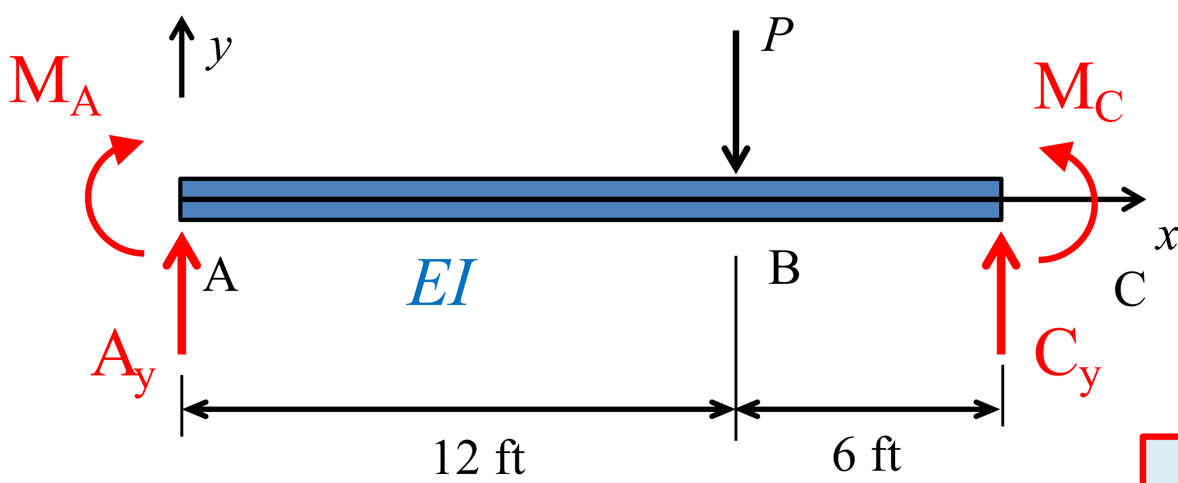


Lab Problem



For the indeterminate beam from Lab 11, subject to the point load at B, find the support reactions at A and C and construct the shear and moment diagram for the beam. Neglect axial force effects.

FBD



Beam is stable

4 unknowns

2 equations of equilibrium

Statically Indeterminate to the 2nd degree

Define Primary Structure and Redundant

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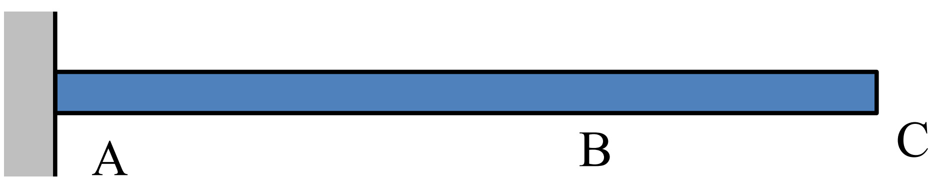
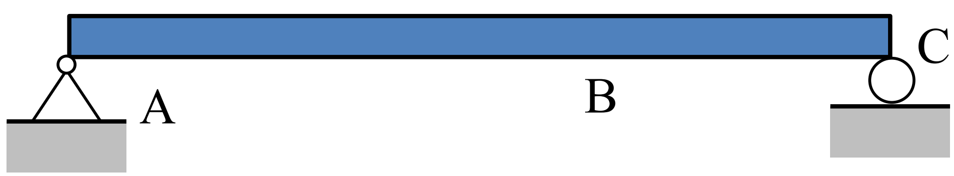
Primary Structure

Redundants



M_C C_y



M_A M_C

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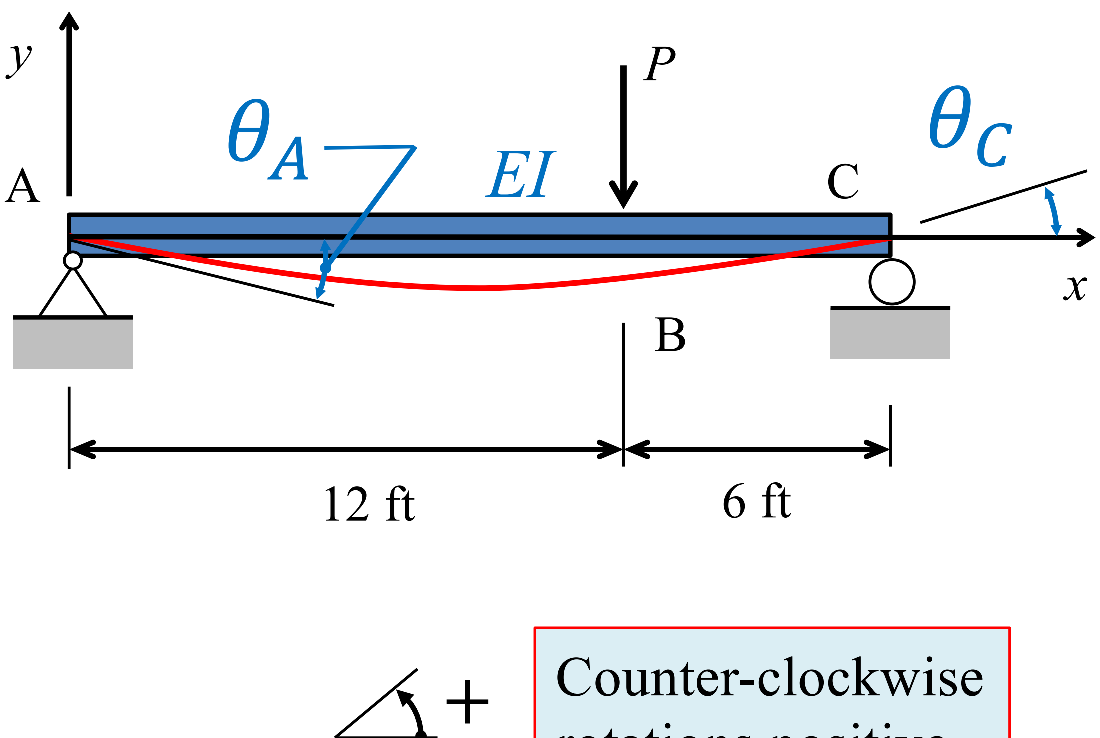


From Tabulated Solutions

$\theta_A = ??$

$\theta_C = ??$

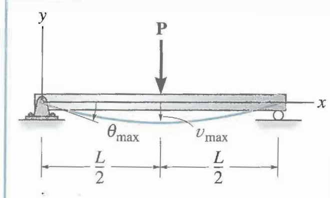
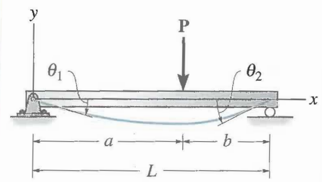
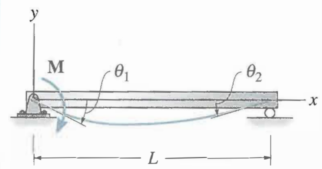
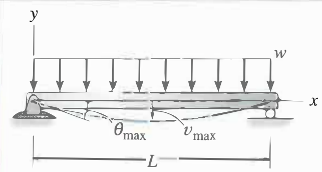
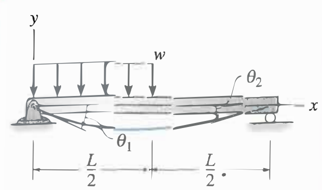
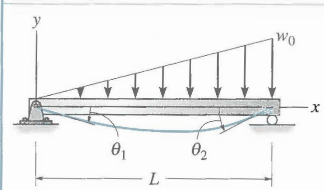
Counter-clockwise rotations positive

Tabulated Solutions

v + ↑

θ (+)

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{Pab(L+b)}{6EIL}$ $\theta_2 = \frac{-Pab(L+a)}{6EIL}$	$v\Big\|_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(-x^2 - b^2 + L^2)$ $0 \le x \le a$
	$\theta_1 = \frac{ML}{3EI}$ $\theta_2 = \frac{-ML}{6EI}$	$v_{max} = \frac{-ML^2}{\sqrt{243}\,EI}$	$v = \frac{-Mx}{6LEI}(x^2 - 3Lx + 2L^2)$ $0 \le x \le L$
	$\theta_{max} = \frac{wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$ $0 \le x \le L$
	$\theta_1 = \frac{3wL^3}{128EI}$ $\theta_2 = \frac{-7wL^3}{384EI}$	$v\Big\|_{x=L/2} = \frac{-5wL^4}{768EI}$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$\theta_1 = \frac{7w_0L^3}{360EI}$ $\theta_2 = \frac{-w_0L^3}{45EI}$		$v = \frac{-w_0x}{360LEI}(3x^4 - 10L^2x^2 + 7L^4)$ $0 \le x \le L$

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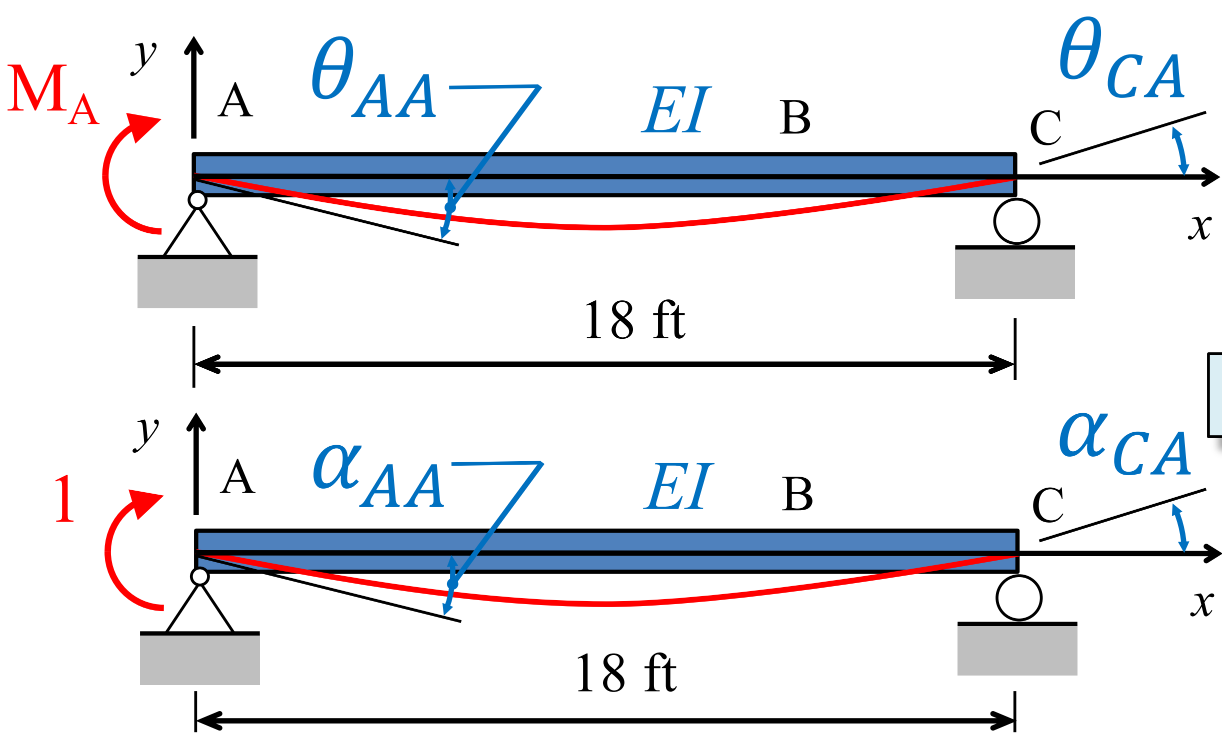


Redundant Problem 1

$$\theta_{AA} = M_A \alpha_{AA}$$

$$\theta_{CA} = M_A \alpha_{CA}$$

From Tabulated Solutions

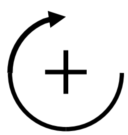
$$\alpha_{AA} = ?$$

$$\alpha_{CA} = ?$$

Tabulated Solutions

v + ↑

θ



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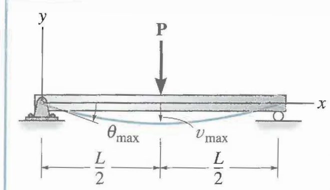
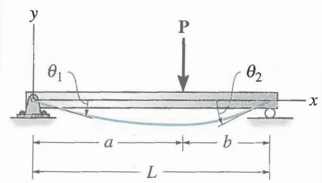
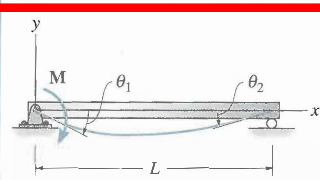
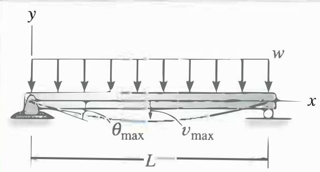
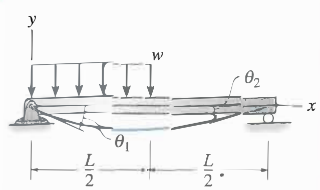
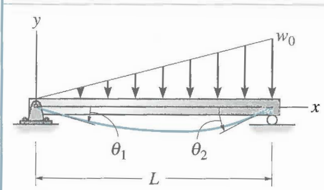
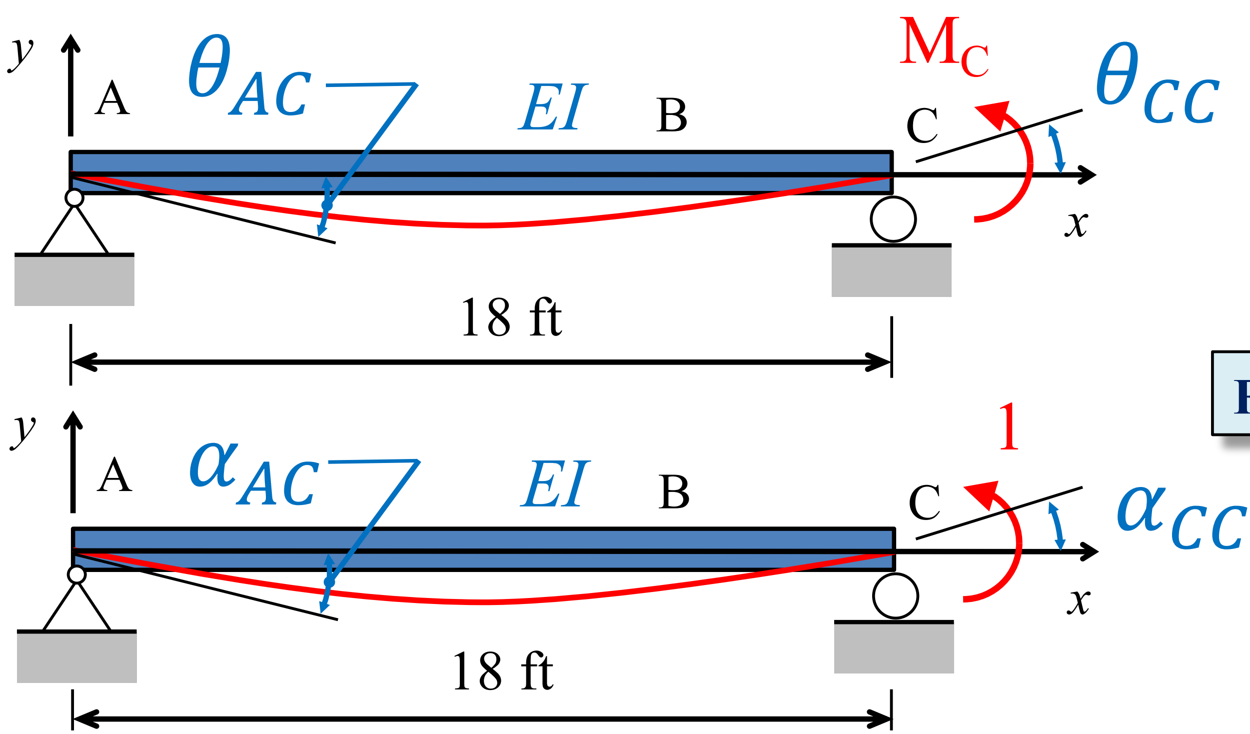
	$\theta_{max} = \frac{PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{Pab(L+b)}{6EIL}$ $\theta_2 = \frac{-Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \frac{-Pba}{6EIL} \cdot (L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(-x^2 - b^2 + L^2)$ $0 \le x \le a$
	$\theta_1 = \frac{ML}{3EI}$ $\theta_2 = \frac{-ML}{6EI}$	$v_{max} = \frac{-ML^2}{\sqrt{243}\,EI}$	$v = \frac{-Mx}{6LEI}(x^2 - 3Lx + 2L^2)$ $0 \le x \le L$
	$\theta_{max} = \frac{wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$ $0 \le x \le L$
	$\theta_1 = \frac{3wL^3}{128EI}$ $\theta_2 = \frac{-7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$\theta_1 = \frac{7w_0L^3}{360EI}$ $\theta_2 = \frac{-w_0L^3}{45EI}$		$v = \frac{-w_0x}{360LEI}(3x^4 - 10L^2x^2 + 7L^4)$ $0 \le x \le L$

Redundant Problem 2



Redundant Problem 2

$$\theta_{AC} = M_C \alpha_{AC}$$

$$\theta_{CC} = M_C \alpha_{CC}$$

From Tabulated Solutions

$$\alpha_{AC} = ?$$

$$\alpha_{CC} = ?$$

Compatibility Equation at Point A

Compatibility at Point A

$$\theta_A + \theta_{AA} + \theta_{AC} = 0$$

Compatibility Equation in terms of Redundant and Flexibility Coefficient

$$\theta_A + M_A \alpha_{AA} + M_C \alpha_{AC} = 0$$

Compatibility Equation at Point C

Compatibility at Point C

$$\theta_C + \theta_{CA} + \theta_{CC} = 0$$

Compatibility Equation in terms of Redundant and Flexibility Coefficient

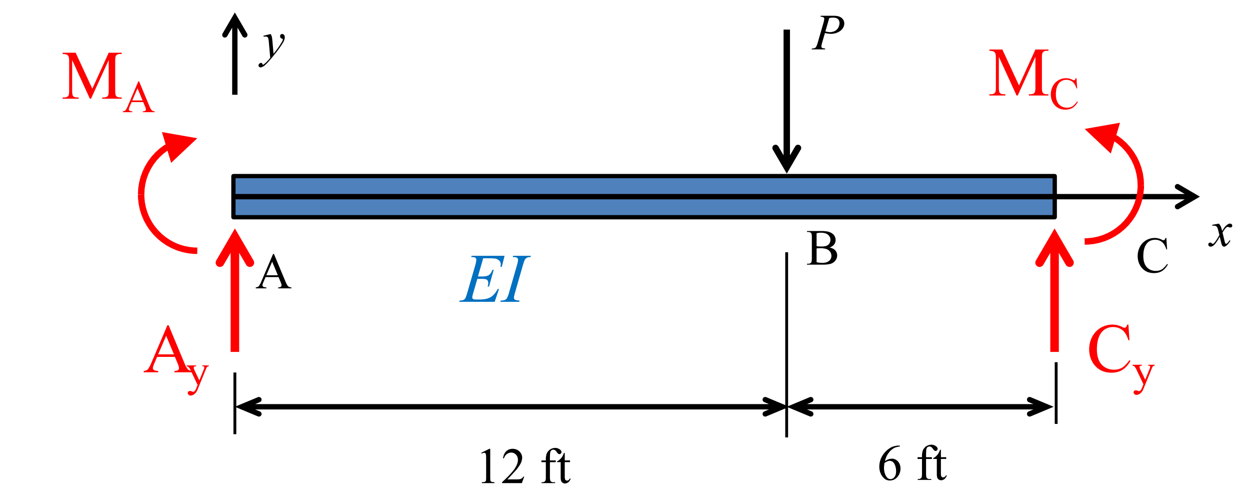
$$\theta_A + M_A \alpha_{CA} + M_C \alpha_{CC} = 0$$

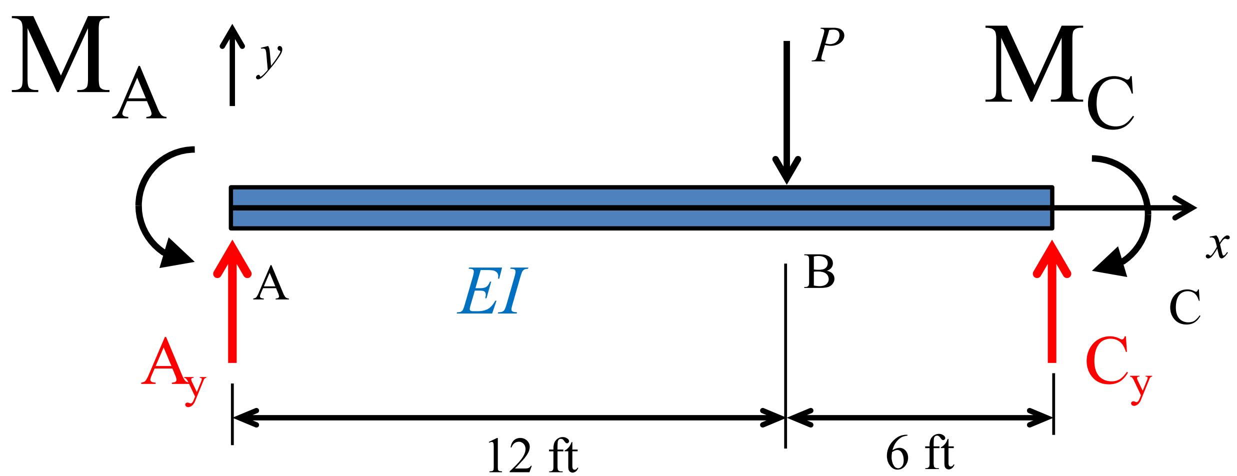
Solve Compatibility Equations for M_A and M_C

$$\theta_A + M_A \alpha_{AA} + M_C \alpha_{AC} = 0$$

$$\theta_A + M_A \alpha_{CA} + M_C \alpha_{CC} = 0$$

Free Body Diagram





Can now use equilibrium equations to find the remaining two unknowns